11:55

Written Homework #5. Math 202 Due Friday, October 23

Name: Solutions Section: ______________

1. Compute the following matrix operations, or explain why they are undefined.

$$A = \begin{bmatrix} 1 & 3 & 0 \\ 4 & 1 & 0 \\ -1 & 0 & 6 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 3 \\ 4 & -1 \end{bmatrix}$$

(a) $2A - B$

$$= 2\begin{bmatrix} 1 & 3 & 0 \\ 4 & 1 & 0 \\ -1 & 0 & 6 \end{bmatrix} + (-1)\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 6 & 0 \\ 8 & 2 & 0 \\ -2 & 0 & 12 \end{bmatrix} + \begin{bmatrix} -1 & 1 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 7 & -1 \\ 8 & 1 & 0 \\ -3 & 0 & 13 \end{bmatrix}$$

(b) $AB - 2I_3$

$$= \begin{bmatrix} 1 & 3 & 0 \\ 4 & 1 & 0 \\ -1 & 0 & 6 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 1 \\ 4 & -3 & 4 \\ 5 & 1 & -7 \end{bmatrix} + \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -5 & 4 \\ 5 & 1 & -9 \end{bmatrix}$$

(c) $2A + 4C$

$$2\begin{bmatrix} 1 & 3 & 0 \\ 4 & 1 & 6 \\ -1 & 0 & 6 \end{bmatrix} + 4\begin{bmatrix} 1 & 3 \\ 4 & -1 \end{bmatrix}$$

DNE

dimensions do not match.

Name: ______________________ Section: ______________________

2. For each of the following, compute the product or explain why it is undefined.

(a) $\underset{3\times 2}{\begin{bmatrix} 1 & 2 \\ 4 & 0 \\ 3 & 0 \end{bmatrix}} \cdot \underset{2\times 3}{\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & -1 \end{bmatrix}} = \begin{bmatrix} 1 & 10 & 1 \\ 4 & 8 & 12 \\ 3 & 6 & 9 \end{bmatrix}_{3\times 3}$

match

(b) $\underset{2\times 3}{\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & -1 \end{bmatrix}} \cdot \underset{2\times 3}{\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{bmatrix}}$ DNE.

do NOT match

(c) $\underset{2\times 3}{\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{bmatrix}} \cdot \underset{3\times 2}{\begin{bmatrix} 1 & 2 \\ 4 & 0 \\ 0 & 3 \end{bmatrix}} = \begin{bmatrix} 9 & 11 \\ 6 & 13 \end{bmatrix}_{2\times 2}$

match

Written Homework #5. Math 202 Due Friday, October 23

Name: ______________________ Section: ______________________

3. For each of the following, compute the inverse of the matrix, or explain why it does not exist.

(a) $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

reduce $[A \mid I_3]$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 2 & 3 & 0 & 0 & 3 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & -1 & -2 & -2 & 1 & 0 \\ 0 & 2 & 2 & 1 & 0 & 1 \end{array}\right] \begin{array}{l} \\ r_2 - 2r_1 \\ r_3 + r_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & -1 & -2 & -2 & 1 & 0 \\ 0 & 0 & -2 & -3 & 2 & 1 \end{array}\right] r_3 + 2r_2 \quad \sim \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 2 & -1 & 0 \\ 0 & 0 & 1 & 3/2 & -1 & -1/2 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & -1/2 & 1 & 1/2 \\ 0 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 3/2 & -1 & -1/2 \end{array}\right] \begin{array}{l} r_1 - r_3 \\ r_2 - 2r_3 \\ \\ \end{array} \qquad 1 - 3/2$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3/2 & -1 & -3/2 \\ 0 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 3/2 & -1 & -1/2 \end{array}\right] r_1 - 2r_2 \qquad \begin{array}{l} -1/2 + 2 = 3/2 \\ 1/2 - 2 = -3/2 \end{array}$$

$$A^{-1} = \begin{bmatrix} 3/2 & -1 & -3/2 \\ -1 & 1 & 1 \\ 3/2 & -1 & -1/2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 3 & -2 & -3 \\ -2 & 2 & 2 \\ 3 & -2 & -1 \end{bmatrix}$$

(b) use A^{-1} to solve $A\vec{x} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$$\vec{x} = A^{-1}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \left(\frac{1}{2}\begin{bmatrix} 3 & -2 & -3 \\ -2 & 2 & 2 \\ 3 & -2 & -1 \end{bmatrix}\right)\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Name: ______________________ Section: ______________________

4. Let $B = \begin{bmatrix} \vec{\mathbf{b}}_1 & \vec{\mathbf{b}}_2 \end{bmatrix}$. Suppose that $\vec{\mathbf{b}}_2 = 3\vec{\mathbf{b}}_1$.

   Prove that that the columns of AB are linearly dependent.

<u>Know</u> $AB = [A\vec{b}_1 \; A\vec{b}_2]$

<u>Know</u> $\vec{b}_2 = 3\vec{b}_1$

so $A\vec{b}_2 = A(3\vec{b}_1) = 3 \cdot A\vec{b}_1$

$A\vec{b}_2 = 3 \cdot A\vec{b}_1$

so $-3A\vec{b}_1 + A\vec{b}_2 = \vec{0}$ ← nontrivial dependence relation between columns of AB.

$\boxed{\text{so columns of } AB \text{ are linearly } \underline{\text{dependent}}}$

5. Use the properties of transpositions to rewrite $(A^T \cdot B^T)^T$. You must show all steps.

$$(A^T \cdot B^T)^T = (B^T)^T \cdot (A^T)^T$$
$$= B \cdot A$$

6. Use the properties of inverses to rewrite $(A^{-1} \cdot B^{-1})^{-1}$. You must show all steps.

$$(A^{-1} \cdot B^{-1})^{-1} = (B^{-1})^{-1}(A^{-1})^{-1}$$
$$= BA.$$